

Problem 4

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.818181 \dots$$

Solution

$$\begin{aligned} 0.818181 \dots &= 0.81 + 0.0081 + 0.000081 + \dots \\ &= \frac{81}{100} + \frac{81}{10\,000} + \frac{81}{1\,000\,000} + \dots \\ &= \sum_{i=1}^{\infty} \frac{81}{10^{2i}} \\ &= \sum_{i=1}^{\infty} \frac{81}{100^i} \\ &= 81 \sum_{i=1}^{\infty} \frac{1}{100^i} \\ &= 81 \sum_{i=1}^{\infty} \left(\frac{1}{100}\right)^i \\ &= 81 \left[-1 + \sum_{i=0}^{\infty} \left(\frac{1}{100}\right)^i \right] \\ &= 81 \left[-1 + \frac{1}{1 - \left(\frac{1}{100}\right)} \right] \\ &= 81 \left(-1 + \frac{100}{99} \right) \\ &= 81 \left(\frac{1}{99} \right) \\ &= \frac{9}{11} \end{aligned}$$